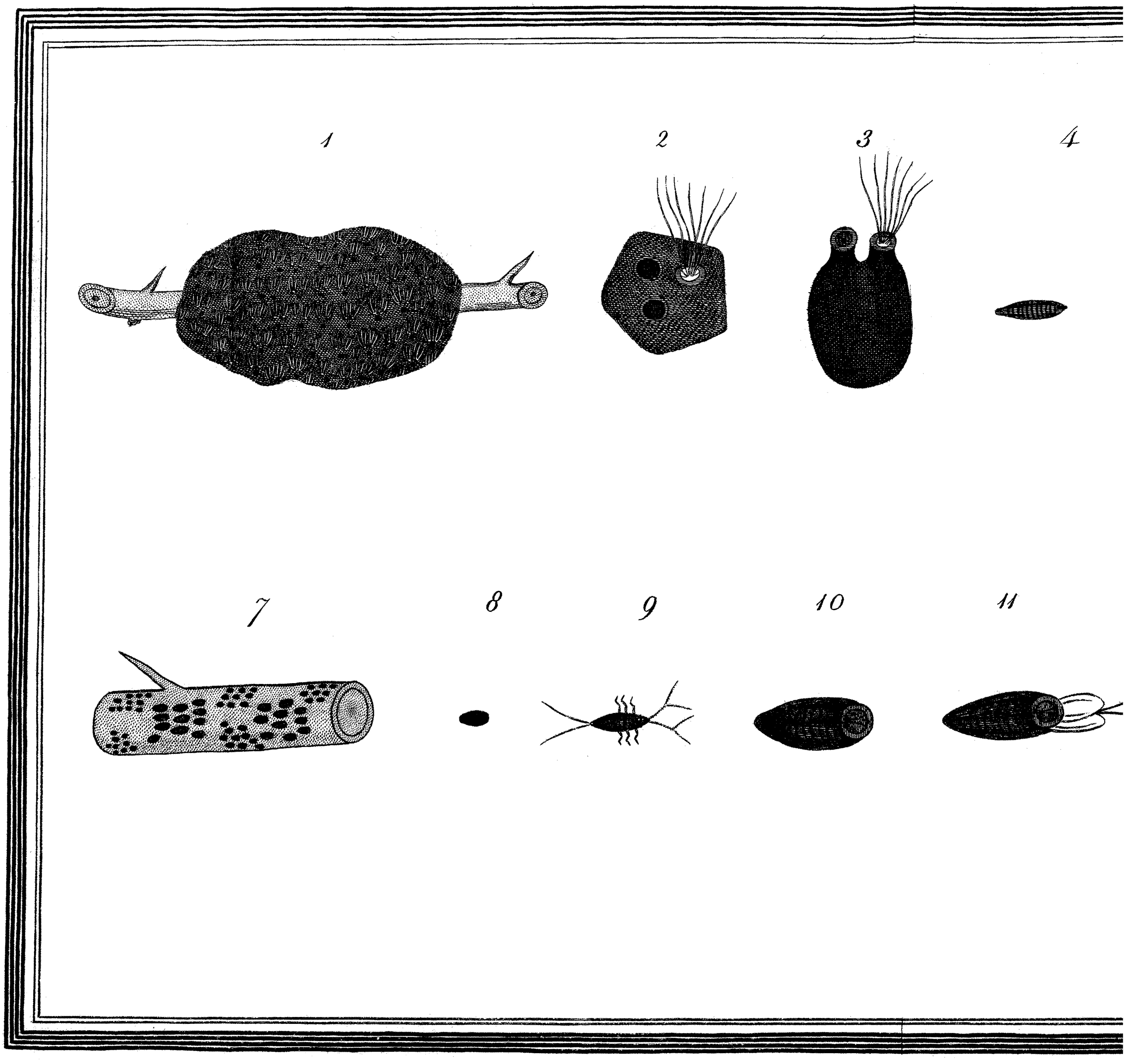
XVI. *The Longitudes of* Dunkirk *and* Paris *from* Greenwich, *deduced from the Triangular Meaſurement in* 1787, 1788, *ſuppoſing the Earth to be an Ellipſoid. By Mr.* Iſaac Dalby; *communicated by* Charles Blagden, *M. D. Sec. R. S.*

Read May 19, 1791.

IN the account of the Trigonometrical Operation in 1787, 1788, which is given in the Philoſophical Tranſactions, Vol. LXXX. after the diſtance of Dunkirk from the meridian of Greenwich has been determined on a parallel to the perpendicular at Greenwich, its longitude is found by ſpherical computation, on a ſuppoſition, that the ſurface of a ſphere nearly coincides with that of the earth in an eaſt and weſt direction, where the operation was performed; and the magnitude of this ſphere, or which amounts to the ſame thing, the value in parts of a degree, &c. of a meaſured arc on its ſurface (for as ſuch the arc between the meridians of Botley Hill and Goudhurſt may be conſidered) has been determined by actual obſervation at two ſtations nearly in the latitude of Dunkirk; and this independent of any hypotheſis which can ſenſibly affect the concluſion. The principles, though not ſtrictly geometrical, admit of little objection; and therefore, as much care was taken in obſerving the angles at theſe ſtations, upon which the directions of the meridians depend, the longitude of Dunkirk (and conſequently that of Paris) as given in the Table, Vol. LXXX. p. 232.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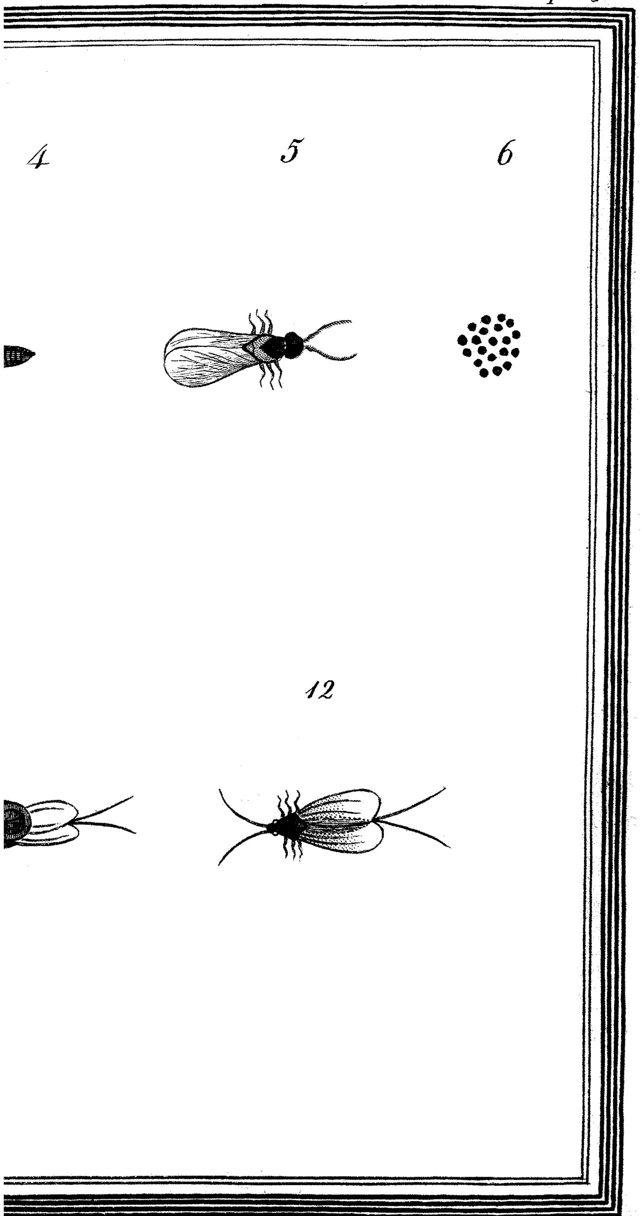

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p. 232. muſt be nearly true, whatever may be the real figure of the earth. But, it may be ſaid, that the arc between the meridians of Botley Hill and Goudhurſt ($17'\frac{1}{3}$) is too ſhort to infer from obſervation the value of the arc between the meridians of Greenwich and Dunkirk (amounting to near a degree and a half), ſufficiently accurate for finding the longitude to great preciſion; becauſe it has been remarked in the Appendix to the ſame Volume, that an error of $1''$, in either of the horizontal angles at the above ſtations, would cauſe a variation of near $6''$ of a degree in the longitude of Dunkirk or Paris.

M. BOUGUER's ſpheroid agreeing nearly with the meridional meaſurements, it was adopted for the purpoſes of latitude. But the degree perpendicular to the meridian in latitude $51° 6' 53''$ is found to be 61248 fathoms (Vol. LXXX. p. 215.) which falls ſhort of M. BOUGUER's degree about 22 fathoms; therefore, ſuppoſing the directions of the meridians to have been very accurately determined, the earth cannot be this ſpheroid, notwithſtanding the ingenious hypotheſis reſpecting the curve of the meridian. But it is alſo well known, that the meaſured degrees of latitude in different places are inconſiſtent with an elliptical meridian: for, ſuppoſe an ellipſoid to be determined with the degrees found at the equator and polar circle, the computed degrees in middle latitudes will be much longer than the meaſured ones, as it is well known; and the whole meridional arc between Greenwich and Paris will, on ſuch an ellipſoid, exceed the meaſured arc by a quantity anſwering to about $21''$ of latitude. It is evident, however, that if we ſuppoſe ſmall errors to have taken place in determining the celeſtial arcs, or differences of latitude in ſome of the operations (for there is little doubt but the terreſtrial menſurations in general have

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been made exact enough), it will be easy to reconcile most of the results to an ellipsoid.

The following computations of the longitude are made on a supposition, that the earth is an ellipsoid, for the purpose of comparing the conclusions with what has been inferred from observation. It will be seen, that the ratio of the axes comes out very near the ratio assigned by Sir ISAAC NEWTON, or 229 to 230. It is determined of such a magnitude, by adhering nearly to the measured arc of the meridian between Greenwich and Paris, deduced from the late operation, that the computed meridional degrees differ but little from the measured ones in five different places in middle latitudes; but the defects at the equator and polar circle are supposed to be nearly equal to each other. This will be seen better by the following comparative view of the measured and computed degrees in the same latitudes.

According to	*Lat.*	*Measured.*	*Computed.*	*Excess or defect in measured arc.*
	° ′	Fath.	Fath.	
M. CONDAMINE, &c.	0 0	60481	60344	+137
MASON and DIXON,	39 12	60628	60682	− 54
BOSCOVICH, &c.	43 0	60725	60738	− 13
CASSINI, &c.	45 0	60778	60768	+ 10
LIESGANIG*,	48 43	60839	60823	+ 16
French and *English*	Arc from latitude 48° 50′ 14″ to 51° 28′ 40″	160656	160662	− 6
MAUPERTUIS, &c.	66 20	61194	61057	+137

In the five comparisons, from latitude 39° 12′ to Greenwich, the greatest error (54 fathoms) answers to about 3″ of the celestial arc: neither of the other four differences amounts

* From the late Gen. ROY's Paper in the Phil. Transf. 1787.

to

to 1″. The determination of M. BECCARIA is not brought into the comparifon, becaufe his meafured degree in latitude 44° 44′ is longer than the meafured one in latitude 45°.

The longitude of Dunkirk on this ellipfoid is found to be 9 m. 29.8 s. in time; and confequently that of Paris 9 m. 20⅓ s., which is about 1½ s. more than that inferred from the value of the meafured arc between Goudhurft and the meridian of Botley Hill; and therefore the fum of the two horizontal angles at thefe ftations would, on this ellipfoid, be only about 4″ *lefs* than thofe found by actual obfervation.

Method of computation.

On an ellipfoid, where the degrees of the meridian at the equator and polar circle are 60481 and 61194 fathoms refpectively, the degree in latitude 50° 9′½ (the middle latitude between Greenwich and Paris) will be 60981 fathoms, exceeding the meafured degree by 140 fathoms (Vol. LXXX. p. 225.); therefore, if each of the former degrees was about 140 fathoms lefs, the computed and meafured arcs in latitude 50° 9′½ would be nearly the fame. But, that they alfo may nearly agree in latitude 45°, let the degrees at the equator, and in latitude 50° 9′½, be taken 60344 and 60844; then, from thefe two degrees, the ratio of the axes will be found as the tangents of the arcs 50° 9′½ and 50° 1′ 35″½; and the femi-axes 3489932 and 3473656 fathoms *.

The

* Determined thus: If right lines are drawn perpendicular to the curve of a *conic fection* to meet the axis, it is known, that *the radii of curvature at the points in the curve from whence thefe lines are drawn, will be as the cubes of thefe lines.* Hence, if PC, GB, EC (Tab. VII. fig. 1.), are perpendicular to the curve, the radii of curvature

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The length of the whole meridional arc between Greenwich and Paris on this ellipſoid is ſix fathoms greater than the meaſured arc; the degree in latitude 48° 43′, 16 fathoms leſs; in latitude 45°, 10 fathoms leſs; in 43°, 13 fathoms greater; and that in latitude 39° 12′, 54 fathoms greater. The degrees at the equator and polar circle are conſiderably leſs than the meaſured ones, conformable to the hypotheſis.

Suppoſe CE, CP (fig. 1.), are the greater and leſs ſemi-axes of the ellipſoid; G Greenwich; PGE its meridian; PD the meridian of Dunkirk; and let GBA be perpendicular to the curve of the meridian at G; then GA will be the ſhorter axis of the elliptical ſection which is the perpendicular to the

at P, G, E, will be as PC^3, GB^3, and $\overline{\frac{PC^2}{CE}}\Big|^3$, becauſe at the point E (or equator) the line ſo drawn will become the radius of curvature itſelf, or $\frac{PC^2}{CE}$. Therefore $GB^3 : \overline{\frac{PC^2}{CE}}\Big|^3$ :: *rad. curv. at* G : *rad. curv. at* E :: *length of a deg. in the lat. of* G : *length of a deg. at* E, the equator. Let the arc ERL be deſcribed with the radius CE; draw CR parallel to GB, RS parallel to PC, and join CK; then, by the nature of the ellipſe, CR (CE) : CK :: GB : *half the parameter*, or $\frac{CP^2}{CE}$; therefore $CE^3 : CK^3 :: GB^3 : \overline{\frac{CP^2}{CE}}\Big|^3$ :: 60844 : 60344 (*ſuppoſing the lat. of the point* G *to be* 50° 9′½), or CE (CR) : CK :: $\overline{60844}|^{\frac{1}{3}} : \overline{60344}|^{\frac{1}{3}}$; but CR : CK :: *ſine* SKC : *ſine* KRC (*co-lat.*); therefore, $\overline{\frac{60844}{60344}}\Big|^{\frac{1}{3}} \times$ *coſine lat.* = *ſine* SKC; hence the angle SCK is given (50° 1′ 35″½); therefore, as *tang.* SCK : *tang. lat.* (SCR) :: SK : SR :: *leſſer ſemi-axis* CP : *greater* CE. And putting $d = 57.295779$, &c. *the degrees in the circular arc which is equal to the radius*) we have $\overline{\frac{tang.\ lat}{tang.\ SCK}}\Big|^2 \times 60344\,d = 3489932$ fathoms the longer ſemi-axis; and $\frac{tang.\ lat.}{tang.\ SCK} \times 60344\,d = 3473656$, the ſhorter.

meridian

meridian at Greenwich, and the angle EBG will be the latitude of Greenwich, or 51° 28′ 40″. Let HO (parallel to GA) be the ſection of the parallel to that perpendicular, paſſing through Dunkirk. Then by the Table, p. 232. (Vol. LXXX.) the arc GH is 152549 feet; but this arc exceeds the real diſtance of the parallels GA, HO, not more than a fathom; therefore this diſtance may be taken = 25424 fathoms. Now the ſections GA, HO, of the ellipſoid being ſimilar, from the known properties of the figure, we ſhall get HO the ſhorter axis of the ſection of the parallel = 6959396, its longer axis = 6979374, and HW = 3531757 fathoms, W being the point where HO cuts the axis PI of the ellipſoid. Hence, if D be Dunkirk, and the arc HD the meaſured arc of the parallel, we have given the length of this arc, or 547058 feet (Table, p. 232.) = 91176 fathoms, and alſo the point W in the leſſer axis of the ſection HO, to determine the angle HWD in the plane of this ſection. But reverting the ſeries which exhibits the length of an elliptical arc in terms of the *abſciſs* and *ordinate*, will be of little uſe in the preſent caſe, where the arc and its chord are very near of the ſame length: For, let HKOL (fig. 2.) be the ſection of the parallel, where HO = 6959396, and KL = 6979374, are the axes; and HW = 3531757, as in *fig.* 1.; alſo, ſuppoſe HS is the radius of curvature at H, or at the middle of HD; then, if we conceive the arc HD to be a right line, or deſcribed with the radius HW, or with HS (3499700) and thence determine the angle SWD from the two ſides SD, SW, and the included angle (the ſupplement of HSD); in either caſe we get the angle HWD the ſame, or 1° 28′ 44″,8 to within 1″. This angle being obtained, the inclination of the planes PHW, PDW (the planes of the meridians of

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Greenwich and Dunkirk, *fig.* 1.), or the longitude of the point D, will be found by the common proportion which in a right-angled ſpherical triangle determines an angle when the legs are given: this will be obvious by conceiving a ſphere (of any magnitude) to be deſcribed about W as a center.

Hence, as *rad.* : *cotang.* angle HWD ($1^\circ\ 28'\ 44''.8$) :: *ſine* angle HWP ($38^\circ\ 31'\ 20''$) : *cotang.* $2^\circ\ 22'\ 26''\frac{1}{2}$, the inclination of the planes of the meridians PH, PD, or longitude of Dunkirk on this ellipſoid. And as the difference of meridians of Paris and Dunkirk is $2'\ 21''.9$ (for this will not be materially affected by different hypotheſes) the longitude of Paris will be 9 m. $20\frac{1}{3}$ s. in time. The longitude of Dunkirk from Paris ($2'\ 21''.9$) is the mean longitude deduced at p. 223. (Vol. LXXX.), which is only $1''.1$ leſs than that given in the *Connoiſſance des Temps*, 1788.

The method of computing the latitude of the point D (was it neceſſary) is thus: as *rad.* : *coſine* DWH :: *coſine* HWP : *coſine* DWP; and ſince the point W in the axis PW is given, and alſo the angle DWP in the plane of the meridian PD (by the foregoing proportion), the point D will be determined by the properties of the ellipſe; which in fact is nothing more than finding the inclination of the vertical at the point D with the given line DW, which inclination added to the angle DWP, gives the co-latitude of the point D. And hence may be evinced the truth of what is advanced at p. 199. (Vol. LXXX.), that *if the value of an arc on a ſpheroid, conſidered as an arc of a great circle perpendicular to the meridian, be given, the longitude may be found by ſpherical computation, but not the latitude.* For conceive the arc HD to be perpendicular to the meridian at H, then the angle HWP would be the co-latitude

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of the point H; and the former proportion would give the longitude of D, whether the figure was a ſphere or ſpheroid; and the angle DWP (found by the latter proportion) would be the co-latitude of D ſuppoſing it a ſphere, in which caſe the point W becomes the center; but this will not hold in a ſpheroid, becauſe DW would not be perpendicular to the meridian at D.

The forgoing method of computing the longitude from the meaſured arc of a parallel on a given ellipſoid (though evidently the direct one), will be tedious, eſpecially when the lengths of the meaſured arcs (GH, HD) are very conſiderable. But when the latitude of the point H is determined from the meaſured arc GH (on the known meridian), and the extent of the other arc (HD), or rather the angle HWD, is not more than two or three degrees, the ſame concluſions, extremely near, may be obtained in the following manner, which is nearly the ſame as the method uſed in computing the longitudes in the Table of General reſults, p. 232. (Vol. LXXX.).

Suppoſe G and D (fig. 1.) to be Greenwich and Dunkirk; PH, PD, their meridians, as before; and let HD (inſtead of its being a parallel to the perpendicular at Greenwich) be an arc of an ellipſe cutting the meridian of Greenwich at right angles, ſuppoſe in the point H. Then the arc GH being = 152549 + 50 feet nearly (becauſe the ellipſe which paſſes through D, and is at right angles to the meridian PG, will fall about 50 feet to the ſouth of the point cut by the parallel), therefore the value of the arc GH, or 25433 fathoms, will, on this ellipſoid, be 25′ 4″.4, and conſequently the angle PWH, or the co-latitude of H, is 38° 56′ 24″.4. Now, the radius of curvature of this perpendicular ellipſe at H, the extremity of its leſſer

 axis,

axis, will be 3499798 fathoms*, which, divided by 57.295779, &c. (the degrees in the circular arc which is equal to the radius), gives 61083 fathoms for a degree on this ellipse, considered as a great circle perpendicular to the meridian at the point H on the ellipsoid; and since the length of this arc (HD) will be nearly the same as that of the parallel, or 91176 fathoms, its value will be 1° 29′ 33″.6 (the arc DH, or rather the angle DWH). Hence, as *rad.* : *cotang.* 1° 29′ 33″.6 (HWD) :: *sine* 38° 56′ 24″.4 (HWP) : *cotang.* 2° 22′ 26″.8, the longitude of D, or Dunkirk, the same as before, very near; hence the longitude of Paris will be 2° 20′ 4″.9. But the same may be obtained from the mean distance of the meridians of Greenwich and Paris, or 537950 feet. See p. 599. in the Appendix to Vol. LXXX.

It appears from the foregoing hypothesis, that the measured degrees of the meridian in middle latitudes will answer nearly on an ellipsoid whose axes are in the ratio assigned by Sir ISAAC NEWTON. But this will receive further confirmation from the fifth ellipsoid in the second Table, p. 232. (Vol. LXXX.), where the near agreement between the computed and measured arc of the meridian between Greenwich and Perpignan (differing but about 52 fathoms in the extent of 8° 46′ 44″) would be somewhat extraordinary, were we certain that the latitude of Perpignan (42° 41′ 56″) is correct; but this is suspected by M. DE LA CAILLE.

* It is not necessary to determine the axes of this ellipse, because when HW is perpendicular to the curve of the meridian, it will (by the nature of the figure) be the radius of curvature of the arc HD at the point H. Hence, if we put v for the *cotang.* and c for the *cosine* of the latitude of the point H, and let a denote the *sine* of an arc whose *tang.* is $\frac{CE}{CP} \times v$; then $\frac{a}{c} \times CE = HW$, by the properties of the ellipse.

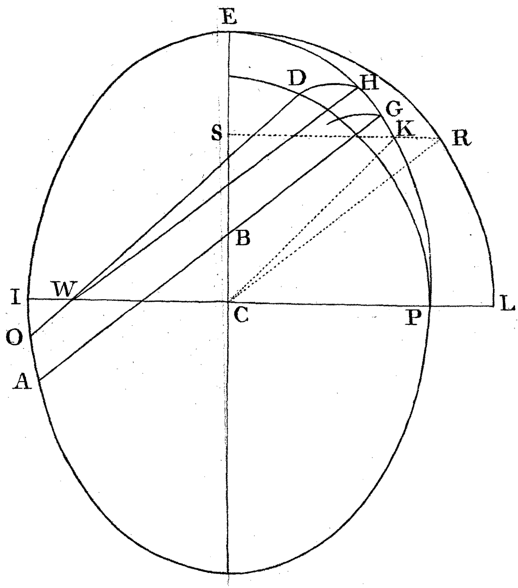

Fig. 1.

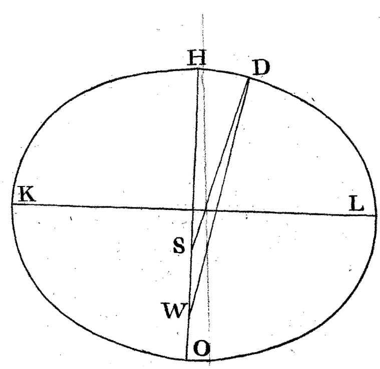

Fig. 2.

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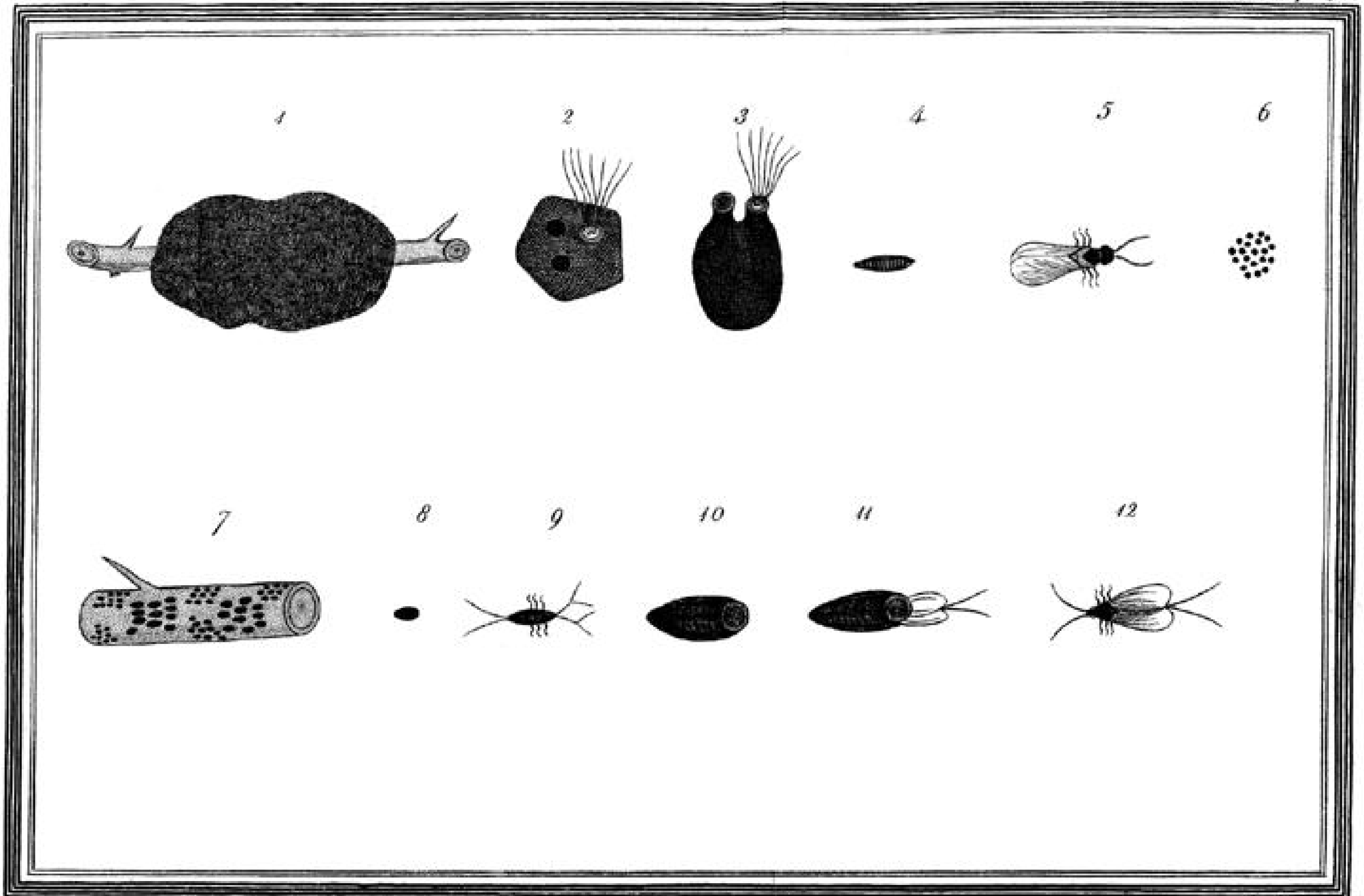
CAILLE. See *Mem. de l'Acad.* 1758. The computed arc, however, between Greenwich and Paris is 19 fathoms longer than the meaſured arc, which anſwers to a little more than 1″ of latitude.

The longitude of Paris on this ellipſoid is 9 m. 20$\frac{4}{10}$ s.

If it be contended, that the operations at the equator and polar circle were as correct as thoſe executed for the like purpoſe in middle latitudes; and that a kind of mean between the extreme reſults ought to be preferred; we ſhall ſtill get an ellipſoid, whoſe axes are nearly as 229 to 230, by taking the degrees at the equator and polar circle each 70 fathoms leſs, and that in latitude 50° 9′$\frac{1}{2}$ as much greater than the meaſured ones; and the longitude of Paris will be found 9 m. 19$\frac{7}{10}$ s. But the computed meridional arc between Greenwich and Paris will exceed the meaſured one by a quantity anſwering to about 11″ of latitude.

It is almoſt needleſs to obſerve, that the longitude of Paris (9 m. 20 s.) deduced by Dr. MASKELYNE from the different reſults found by aſtronomical obſervations (Phil. Tranſ. 1787) agrees to leſs than half a ſecond with either of the above determinations.

N. B. In the Table of General Reſults, p. 232. Vol. LXXX. *read* 18′ 46″ for the longitude of Wrotham Hill; this ſhould have been corrected in the Appendix.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